14TH INTERNATIONAL YOUNG RESEARCHER WORKSHOP ON GEOMETRY, MECHANICS AND CONTROL
GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

DECEMBER 16–18, 2019

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mini-course

Some application of perturbative theory to the N-body problem

As a breakthrough application of Kolmogorov's theorem on the conservation of invariant torus, V.I.Arnold, in the 60s, formulated a theorem on the stability of planetary motions for infinite times [11, 2]. The proof of that theorem relied on two main ingredients: a careful form Kolmogorov's theorem, suited to degenerate (i.e., "super—integrable") systems and a detailed analysis of the symplectic structure of the phase spase of the problem. The former aspect was fully and successfully solved by Arnold; the latter required about 50 years to be completely understood. The proof given in [16, 6] relies on a system of Poincaré -like system of canonical coordinates well suited to the SO(3) invariance of the system. Since then, other systems of coordinates have been discovered, leading to different results; e.g., [18]. In these lectures, we shall highlight the importance of canonical coordinates in the perturbative approach to the N-body problem.

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